1.

obecně: $a \cdot (x - x_1) = 0$, $a \neq 0$, x_1 je kořen

a) např: $x - (-\frac{3}{4}) = 0$

$x = -\frac{3}{4}$

b) např. $6 - 3x = 3(2 - x) \rightarrow 6 - 3x = 6 - 3x$

$0x = 0 \ldots \forall x \in (-\infty, \infty)$

c) např. $3x - 4 = 3x + 4$

$0x = 8 \rightarrow K = \emptyset$

2

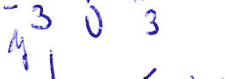
a) $|x| = 3$

1) $x \geq 0 \ldots x = 3$

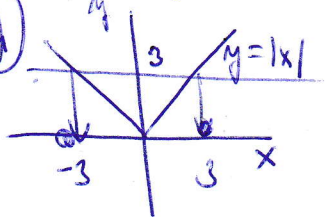
$x < 0 \ldots -x = 3 \Rightarrow x = -3$

$K = \{-3, 3\}$

b) 1) +3 +3 (3 doleva, 3 doprava)



2)



b) $|x - 2| = 5$

1) $x - 2 \geq 0$

$x \geq 2 \ldots x - 2 = 5$

$x = 7$ OK

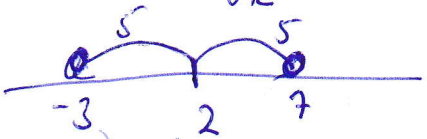
$x - 2 \leq 0$

$x \leq 2 \ldots 2 - x = 5$

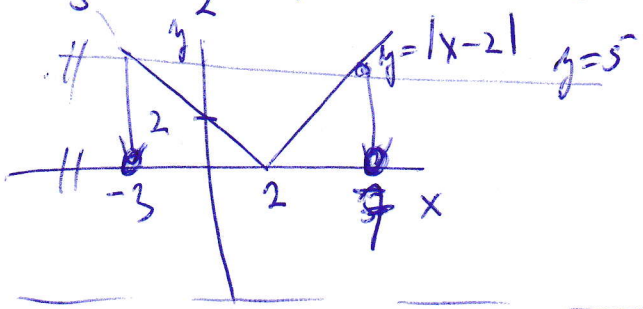
$x = -3$ OK

$K = \{-3, 7\}$

2)



3)



c) $|x - 3| = |x + 4|$

2)

	$(-\infty, -4\rangle$	$\langle -4, 3\rangle$	$\langle 3, \infty)$
$\|x-3\|$	-	-	+
$\|x+4\|$	-	+	+

$(-\infty, -4\rangle \quad -x + 3 = -x - 4$

$\emptyset$

$\langle -4, 3\rangle \quad -x + 3 = x + 4$

$-1 = 2x$

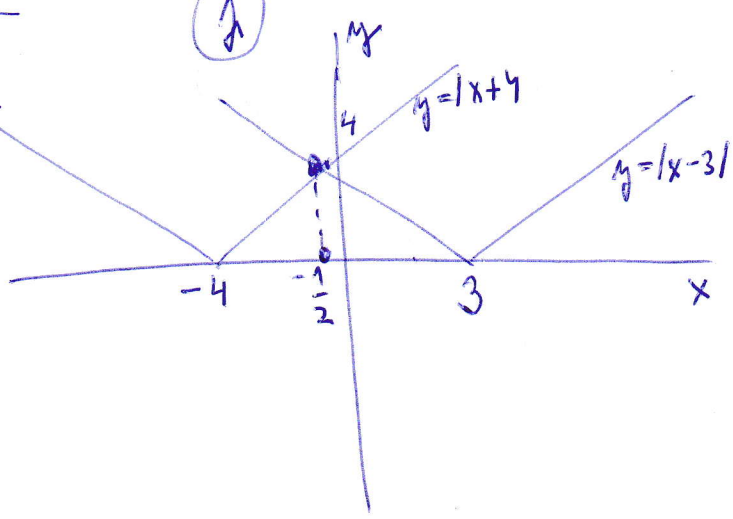
$x = -\frac{1}{2}$ OK

$K = \{-\frac{1}{2}\}$

$\langle 3, \infty) \quad x - 3 = x - 4$

$\emptyset$

3)



(3)

a) $(5x+2)(5-2x)=0$

$5x+2=0$ nebo $5-2x=0$

$x=-\frac{2}{5}$ $x=\frac{5}{2}$

$K=\{-\frac{2}{5}, \frac{5}{2}\}$

b) $x^2-7=0$

$x^2-(\sqrt{7})^2=0$

$(x-\sqrt{7})(x+\sqrt{7})=0$

$K=\{-\sqrt{7}, \sqrt{7}\}$

nebo: $x^2=7$

$|x|=\sqrt{7}$

$x=\pm\sqrt{7}$

c) $4x^2-8x-32=0 \quad |:4$

$x^2-2x-8=0$

diskriminant: $D=4+32=36$

$x_{1,2}=\frac{2\pm\sqrt{36}}{2}=\begin{cases}\frac{2+6}{2}=4\\ \frac{2-6}{2}=-2\end{cases}$

$K=\{-2, 4\}$

d) $\frac{x^2-8x}{x}=0 \quad \dots \quad x\neq 0$

zlomek je nula $\Leftrightarrow$ čitatel je 0

$x^2-8x=0$

$x(x-8)=0$

$x=0$ nebo $x-8=0$

$x=8$

$\rightarrow x=0$ nelze

e) $x^2=|x+2|$

	$(-\infty, -2\rangle$	$\langle -2, \infty)$
$\|x+2\|$	$-$	$+$
	$x^2=-x-2$	$x^2=x+2$
	$x^2+x+2=0$	$x^2-x-2=0$
	$D=1-8<0$	$D=1+8=9$
	$\emptyset$	$x_{1,2}=\frac{1\pm\sqrt{9}}{2}$
		$x_1=2, \quad x_2=-1$
		$x_1\in\langle -2, \infty)$ ✓
		$x_2\in\langle -2, \infty)$ ✓

$K=\{-1, 2\}$

f) $\sqrt{x+3}-1=\sqrt{x-2} \quad /^2$

$x+3-2\sqrt{x+3}+1=x-2$

$-2\sqrt{x+3}=-6 \quad |:(-2)$

$\sqrt{x+3}=3 \quad /^2$

$x+3=9$

$x=6$

! umocnění na 2. je nekvivalentní úprava $\Rightarrow$ zkouška je nutnou součástí řešení

Zk. $L=\sqrt{6+3}-1=3-1=2$

$P=\sqrt{6-2}=2$ $\quad L=P$ ✓

$K=\{6\}$

(3)

a) $\sqrt{x^2+3x} = x^2+3x-2$

Substituce: $t = \sqrt{x^2+3x} \rightarrow t^2 = x^2+3x$

$t = t^2-2$

$0 = t^2-t-2$

$t_1 = 2 \qquad t_2 = -1$

$\sqrt{x^2+3x} = 2 \,/^2$ | $\sqrt{x^2+3x} = -1$

$x^2+3x-4=0$ | $\emptyset$

$D = 9+16 = 25$

$x_1 = \frac{-3-5}{2} = -4$

$x_2 = \frac{-3+5}{2} = 1$

$K = \{-4, 1\}$

(4) $x_1 = 3, \quad x_2 = -7$

$a(x-x_1)(x-x_2) = 0$

$a(x-3)(x+7) = 0$

normovaný tvar: $a = 1$

$(x-3)(x+7) = 0$

$x^2+4x-21 = 0$

⑤ $x^2 - 5x + 1 = 0$

$x_1 \cdot x_2 = 1$

$x_1 + x_2 = 5 \rightarrow (x_1 + x_2)^2 = 25$

$(x_1 + x_2)^2$

$(x_1 - x_2)^2 = x_1^2 - 2x_1x_2 + x_2^2 = x_1^2 + 2x_1x_2 + x_2^2 - 4x_1x_2 =$

$= (x_1 + x_2)^2 - 4x_1x_2 =$

$= 25 - 4 \cdot 1 = 21$

Hledaná rovnice $a \cdot (x - \text{první kořen})(x - \text{druhý kořen}) = 0, \; a \neq 0$

$a \cdot (x - (x_1 + x_2)^2)(x - (x_1 - x_2)^2) = 0$

$a \cdot (x - 25) \cdot (x - 21) = 0, \; a \neq 0$

⑥

a) $x + 2y = 4$

$3x - y = 5 \mid \cdot 2$

$x + 2y = 4$

$6x - 2y = 10$

$7x = 14$

$x = 2$

$y = 3x - 5$

$y = 1$

řešením je uspořádaná dvojice $[x, y] = [2, 1]$

sčítací metoda

b) $\frac{1}{x} + \frac{3}{y} = 5 \mid \cdot (-2)$

$\frac{2}{x} - \frac{6}{y} = 6$

$-\frac{2}{x} - \frac{6}{y} = -10$

$\frac{2}{x} - \frac{6}{y} = 6$

$-\frac{12}{y} = -4$

$y = 3$

$\frac{2}{x} - \frac{6}{3} = 6$

$\frac{2}{x} = 8$

$\frac{x}{2} = \frac{1}{8}$

$x = \frac{1}{4}$

$K = \{[\frac{1}{4}, 3]\}$

sčítací metoda

c) $x^2 - 2y^2 = 8$

$x + 3y = 2 \Rightarrow x = 2 - 3y$

$(2 - 3y)^2 - 2y^2 = 8$

$4 - 12y + 9y^2 - 2y^2 = 8$

$7y^2 - 12 - 4 = 0$

$D = 144 + 16 \cdot 7 = 256$

$y_{1,2} = \frac{12 \pm 16}{14} = \begin{cases} 2 \\ -\frac{2}{7} \end{cases}$

$x_1 = 2 - 3y_1 = 2 - 6 = -4$

$x_2 = 2 - 3y_2 = 2 + \frac{6}{7} = \frac{20}{7}$

$K = \{[-4, 2], [\frac{20}{7}, -\frac{2}{7}]\}$