$\boxed{\text{EXP. A LOG.}}$

1a) $8^{4x-3} = 16^{5}$

$2^{3(4x-3)} = (2^{4})^{5}$

$2^{12x-9} = 2^{20}$

$12x - 9 = 20$

$\underline{\underline{x = \frac{29}{12}}}$

1b) $4^{x-1} + 4^{x-2} + 4^{x-3} = 42$

$\frac{1}{4} \cdot 4^{x} + \frac{1}{4^2} \cdot 4^{x} + \frac{1}{4^3} \cdot 4^{x} = 42$

$4^{x}\left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3}\right) = 42$

$4^{x}\left(\frac{4^2+4+1}{4^3}\right) = 42$

$4^{x} = \frac{42 \cdot 4^3}{21}$

$4^{x} = 2 \cdot (2^2)^3$

$\cancel{(2^2)^x =}$

$2^{2x} = 2^{1+6}$

$2x = 7$

$\underline{\underline{x = \frac{7}{2}}}$

1c) $3^{2x} - 10 \cdot 3^{x} = -9$ substituce: $3^{x} = a$

$(3^{x})^2 - 10 \cdot 3^{x} + 9 = 0$

$a^2 - 10a + 9 = 0$

$a_1 = 1$ $a_2 = 9$

$3^{x} = 1$ $3^{x} = 9$

$3^{x} = 3^{0}$ $3^{x} = 3^{2}$

$\underline{\underline{x = 0}}$ $\underline{\underline{x = 2}}$

1d) $0{,}5^{x} = 4\sqrt{2}$

$\left(\frac{1}{2}\right)^{x} = 2^{2} \cdot 2^{\frac{1}{2}}$

$2^{-x} = 2^{2+\frac{1}{2}}$

$\underline{\underline{x = -\frac{5}{2}}}$

1e) $2^{4x+2} - 9^{2x+1} > 0$

$2^{4x} \cdot 2^{2} > 9^{2x} \cdot 9$

$4 \cdot 2^{4x} > 9 \cdot 3^{4x}$

$\frac{4}{9} > \left(\frac{3}{2}\right)^{4x}$

$\left(\frac{2}{3}\right)^{2} > \left(\frac{3}{2}\right)^{4x}$ nebo $\left(\frac{2}{3}\right)^{2} > \left(\frac{3}{2}\right)^{4x}$

$\left(\frac{3}{2}\right)^{-2} > \left(\frac{3}{2}\right)^{4x}$ ... základ > 1 $\left(\frac{2}{3}\right)^{2} > \left(\frac{2}{3}\right)^{-4x}$ ... základ < 1

$-2 > 4x$ $2 < -4x \;|: (-4)$ ...

$x < -\frac{1}{2}$ $-\frac{2}{4} > x$

$x < -\frac{1}{2}$

$\underline{\underline{\forall x \in \left(-\infty, -\frac{1}{2}\right)}}$

1f) $\log_{16} x = 2 \rightarrow x > 0$

$16^{2} = x$

$\underline{\underline{x = 256}}$

U log. rovnic a nerovnic je třeba napsat a vyřešit podmínky (nebo u rovnic udělat zkoušku)

1g) $\log(x^{2}) = 6 \rightarrow x^{2} > 0 \rightarrow x \neq 0$

$x^{2} = 10^{6}$

$|x| = 10^{3}$

$\underline{\underline{x = \pm 10^{3}}}$

1h) $1 + \log x^3 = \frac{10}{\log x} \quad | \cdot \log x \quad \ldots \quad x>0$ substituce: $t = \log x$

$$\log x + 3\log x \cdot \log x - 10 = 0$$

$$3(\log x)^2 + \log x - 10 = 0$$

$$3t^2 + t - 10 = 0$$

$t_1 = -2$ $\qquad$ $t_2 = \frac{5}{3}$

$\log x = -2$ $\qquad$ $\log x = \frac{5}{3}$

$x = 10^{-2}$ $\qquad$ $\underline{\underline{x = 10^{\frac{5}{3}}}}$

$\underline{\underline{x = 0{,}01}}$

1i) $x^2 e^x + 6e^x > 5x e^x$

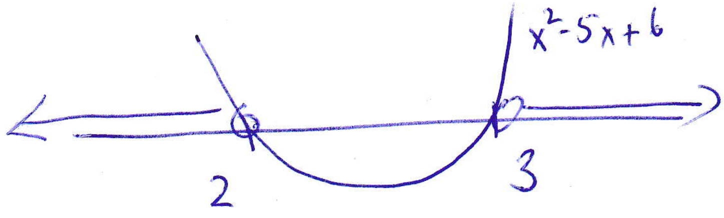
$$\cancel{e^x(x^2+6) > e^x \cdot 5x}$$

$$e^x(x^2+6) > e^x \cdot 5x \quad | : e^x \ (\text{je vždy} \neq 0)$$

$$x^2 + 6 > 5x$$

$$x^2 - 5x + 6 > 0$$

$x^2 - 5x + 6 = 0 \rightarrow x_1 = 2 \qquad x_2 = 3$



$\underline{\underline{\forall x \in (-\infty, 2) \cup (3, \infty)}}$

2 a) $2^{\log x} \cdot 3^{\log y} = \sqrt{54}$

$\log x + \log y = 2$

substituce:

$a = \log x$

$b = \log y$

$2^a \cdot 3^b = \sqrt{54}$

$a + b = 2 \Rightarrow a = 2 - b$

$2^{(2-b)} \cdot 3^b = \sqrt{54}$

$4 \cdot 2^{-b} \cdot 3^b = \sqrt{54}$

$4 \cdot \left(\frac{3}{2}\right)^b = \sqrt{54}$

$\left(\frac{3}{2}\right)^b = \frac{\sqrt{54}}{4}$

$\left(\frac{3}{2}\right)^b = \sqrt{\frac{54}{16}}$

$\left(\frac{3}{2}\right)^b = \sqrt{\frac{27}{8}}$

$\left(\frac{3}{2}\right)^b = \sqrt{\left(\frac{3}{2}\right)^3}$

$\left(\frac{3}{2}\right)^b = \left(\frac{3}{2}\right)^{\frac{3}{2}}$

$b = \frac{3}{2}$

$a = 2 - \frac{3}{2}$

$a = \frac{1}{2}$

$a = \log x = \frac{1}{2}$

$x = \sqrt{10}$

$b = \log y = \frac{3}{2}$

$y = \sqrt{10^3}$

$[x; y] = [\sqrt{10}; \sqrt{10^3}]$

2b) $5^{x+1}+3^{y+1}=26$ (v Z)

$9(5^{2x}+3^{2y})=226$

Substituce:

$a=5^{x}$; $a^2=5^{2x}$

$b=3^{y}$; $b^2=3^{2y}$

$5\cdot 5^{x}+3\cdot 3^{y}=26$

$9\cdot 5^{2x}+9\cdot 3^{2y}=226$

$5a+3b=26 \rightarrow b=\frac{26-5a}{3}$

$9a^2+9b^2=226$

$9a^2+\cancel{9}\cdot\frac{(26-5a)^2}{\cancel{9}}=226$

$9a^2+25a^2-260a+676-226=0$

$34a^2-260a+450=0$

$a_1=\frac{45}{17}$ | $a_2=5$

$b_1=\frac{26-5\cdot\frac{45}{17}}{3}$ | $b_2=\frac{26-5\cdot 5}{3}$

$b_1=\frac{217}{51}$ | $b_2=\frac{1}{3}$

$5^{x}=\frac{45}{17} \rightarrow x=\frac{\log\frac{45}{17}}{\log 5}$ | $5^{x}=5$, $x=1$

$3^{y}=\frac{217}{51} \Rightarrow y=\frac{\log\frac{217}{51}}{\log 3}$ | $3^{y}=\frac{1}{3}$, $3^{y}=3^{-1}$, $y=-1$

$$\boxed{[x,y]=\left[\frac{\log\frac{45}{17}}{\log 5};\frac{\log\frac{217}{51}}{\log 3}\right] \quad [x,y]=[1;-1]}$$

3

GONIOM.

3a) $2\cos\left(2x+\frac{\pi}{6}\right) = -1$ substituce: $t = 2x + \frac{\pi}{6}$

$\cos t = -\frac{1}{2}$ < II. kvadrant, III. kvadrant } tam je kosinus záporný

$\cos t_0 = \frac{1}{2}$

$t_0 = \frac{\pi}{3}$

II. kvadrant

$t = \left(\pi - \frac{\pi}{3}\right) + 2k\pi$

$t = \frac{2\pi}{3} + 2k\pi$

$2x + \frac{\pi}{6} = \frac{2\pi}{3} + 2k\pi$

$2x = \frac{4\pi - \pi}{6} + 2k\pi$

$x = \frac{3\pi}{12} + k\pi$

$\underline{x = \frac{\pi}{4} + k\pi}$

III. kvadrant

$t = \left(\pi + \frac{\pi}{3}\right) + 2k\pi$

$t = \frac{4\pi}{3} + 2k\pi$

$2x + \frac{\pi}{6} = \frac{4\pi}{3} + 2k\pi$

$2x = \frac{8\pi - \pi}{6} + 2k\pi$

$x = \frac{7\pi}{12} + k\pi$

$K = \left\{ \frac{\pi}{4} + k\pi;\ \frac{7\pi}{12} + k\pi;\ k \in \mathbb{Z} \right\}$

3b) $\sin x = \sin 2x$

$\sin x = 2\sin x \cos x$

$2\sin x \cos x - \sin x = 0$

$\sin x\,(2\cos x - 1) = 0$

$\sin x = 0$

$x = k\pi,\ k \in \mathbb{Z}$

$2\cos x = 1$

$\cos x = \frac{1}{2}$ < I., IV.

$x_0 = \frac{\pi}{3}$

I. kvadrant

$x = \frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$

IV. kvadrant

$x = \left(2\pi - \frac{\pi}{3}\right) + 2k\pi$

$x = \frac{5\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$

$K = \left\{ k\pi;\ \frac{\pi}{3} + 2k\pi;\ \frac{5\pi}{3} + 2k\pi;\ k \in \mathbb{Z} \right\}$

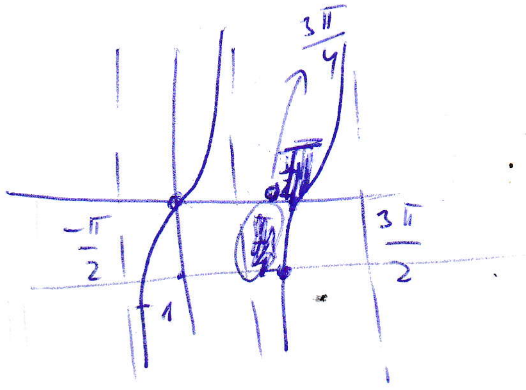
3c) $\sin x = -\cos x$

pro $\cos x \neq 0$:

$$\frac{-\sin x}{\cos x} = 1$$

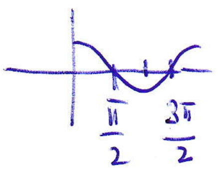
$$\operatorname{tg} x = -1$$

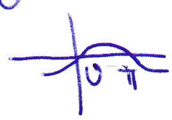
$$\underline{x = \frac{3\pi}{4} + k\pi,\ k \in \mathbb{Z}}$$



pro $\cos x = 0 \Rightarrow \sin x = 0$

$$\underbrace{x = \frac{\pi}{2} + k\pi \qquad x = \pi + k\pi}_{\emptyset}$$





$$\underline{K = \left\{ \frac{3\pi}{4} + k\pi;\ k \in \mathbb{Z} \right\}}$$

3d) $\cos 2x - 2 = \cos x$

$\cos^2 x - \sin^2 x - 2 = \cos x$

$\cos^2 x - (1 - \cos^2 x) - 2 = \cos x$

$2\cos^2 x - \cos x - 3 = 0$

substituce: $t = \cos x$

$2t^2 - t - 3 = 0$

$t_1 = -1$

$\cos x = -1$

$$\underline{x = \pi + 2k\pi,\ k \in \mathbb{Z}}$$

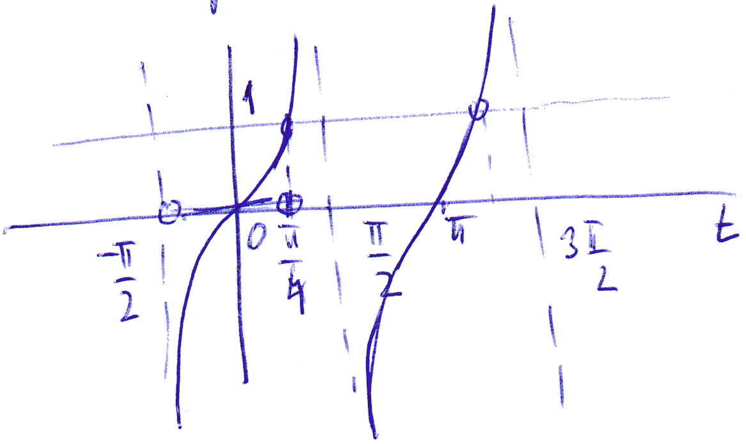
$t_2 = \frac{3}{2}$

$\cos x = \frac{3}{2}$

$\emptyset$ (obor hodnot funkce $y = \cos x$ je $\langle -1, 1 \rangle$)

3e) $tg(4x-\pi) < 1$ subst. $t = 4x-\pi$

$tg\, t < 1$



$t \in \left(-\frac{\pi}{2}+k\pi;\ \frac{\pi}{4}+k\pi\right)$

$-\frac{\pi}{2}+k\pi < t < \frac{\pi}{4}+k\pi$

$-\frac{\pi}{2}+k\pi < 4x-\pi < \frac{\pi}{4}+k\pi$

$-\frac{\pi}{2}+k\pi < 4x-\pi$

$\frac{\pi}{2}+k\pi < 4x$

$x > \frac{\pi}{8}+k\frac{\pi}{4}$

$4x-\pi < \frac{\pi}{4}+k\pi$

$4x < \frac{5\pi}{4}+k\pi$

$x < \frac{5\pi}{16}+k\frac{\pi}{4}$

$$\frac{\pi}{8}+k\frac{\pi}{4} < x < \frac{5\pi}{16}+k\frac{\pi}{4}, \quad k \in \mathbb{Z}$$

3 f)

$$\frac{2\sin^2 x + \sin 2x}{\cos^2 x} \geqq 0$$

$\cos^2 x$ ↳ vždy > 0

$\Rightarrow$ stačí vyřešit:

$$2\sin^2 x + \sin 2x \geqq 0$$

$$2\sin^2 x + 2\sin x \cos x \geqq 0$$

$$2\sin x\,(\sin x + \cos x) \geqq 0$$

1) $\sin x \geqq 0 \quad \wedge \quad \sin x + \cos x \geqq 0$

$x \in \langle 0 + 2k\pi ; \pi + 2k\pi \rangle \quad \wedge \quad \operatorname{tg} x \geqq -1$

$x \in \langle \frac{3\pi}{4} + k\pi ; \frac{3\pi}{2} + k\pi ; \cancel{k \in \mathbb{Z}})$

$x \in \langle \frac{3\pi}{4} + 2k\pi ; \pi + 2k\pi \rangle$

2) $\sin x \leqq 0 \quad \wedge \quad \sin x + \cos x \leqq 0$

$x \in \langle \pi + 2k\pi ; 2\pi + 2k\pi \rangle \quad \wedge \quad \operatorname{tg} x \leqq -1$

$x \in (-\frac{\pi}{2} + k\pi ; -\frac{\pi}{4} + k\pi \rangle$

⋮