h) $f(x) = \sqrt{x^2 - x^4}$

$$x^2 - x^4 \geq 0$$

$$\underbrace{x^2}_{\geq 0} \cdot (1 - x^2) \geq 0$$

$x \in \langle -1, 1 \rangle$

$D(f) = \langle -1, 1 \rangle$

f je spojitá na $D(f)$, f je sudá

$f(-1) = f(1) = 0$, $f(0) = 0$

$$f'(x) = \frac{1}{2 \cdot \sqrt{x^2 - x^4}} \cdot (2x - 4x^3) = \frac{x - 2x^3}{\sqrt{x^2 - x^4}} = \frac{x \cdot (1 - 2x^2)}{\sqrt{x^2 - x^4}}$$

$D(f') = (-1, 1) \setminus \{0\}$

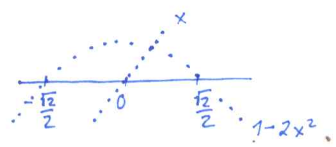
$$f'(x) = 0 \iff 1 - 2x^2 = 0$$

$$(1 - \sqrt{2}\,x) \cdot (1 + \sqrt{2}\,x) = 0$$

$$x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \quad \vee \quad x = -\frac{\sqrt{2}}{2}$$

kritické body : $x = 0$ ($f'(0) \nexists$)

$x = \frac{\sqrt{2}}{2}$, $x = -\frac{\sqrt{2}}{2}$



$\sqrt{x^2 - x^4} > 0 \quad \forall x \in D(f')$

znaménko f' :

	-1		$-\frac{\sqrt{2}}{2}$		0		$\frac{\sqrt{2}}{2}$		1
f'		$\oplus$		$\ominus$		$\oplus$		$\ominus$	
f		$\nearrow$		$\searrow$		$\nearrow$		$\searrow$	

$\Rightarrow$ $x = \pm \frac{\sqrt{2}}{2}$ lok. maxima , $f(\pm \frac{\sqrt{2}}{2}) = \frac{1}{2}$

$x = 0$ lok. minimum , $f(0) = 0$

$$\lim_{x \to -1+} f'(x) = \lim_{x \to -1+} \frac{x - 2x^3}{\sqrt{x^2 - x^4}} = \text{"}\frac{+1}{0+}\text{"} = \underline{\underline{+\infty}}$$

$$\lim_{x \to 1-} f(x) = \text{"}\frac{-1}{0+}\text{"} = \underline{\underline{-\infty}}$$

($\Rightarrow$ f se v krajních bodech chová jako odmocnina)

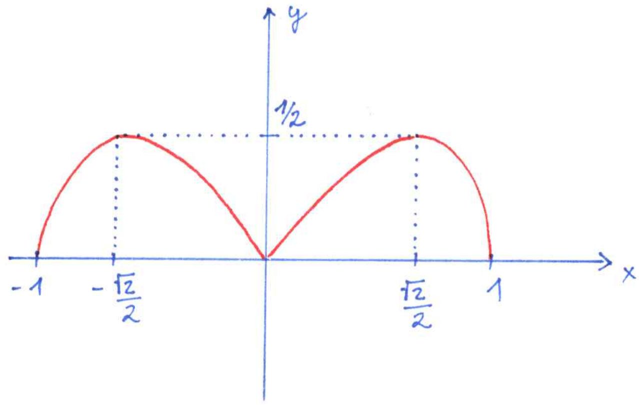
$$f'(0+) = \lim_{x \to 0+} f'(x) = \lim_{x \to 0+} \frac{x - 2x^3}{\sqrt{x^2 - x^4}} = \text{"}\frac{0}{0}\text{"} = \lim_{x \to 0+} \frac{x \cdot (1 - 2x^2)}{|x| \cdot \sqrt{1 - x^2}} = \lim_{x \to 0+} \frac{1 - 2x^2}{\sqrt{1 - x^2}} = \underline{\underline{1}}$$

$\Rightarrow |x| = x$

$$f'(0-) = \lim_{x \to 0-} \frac{x \cdot (1 - 2x^2)}{|x| \cdot \sqrt{1 - x^2}} = \lim_{x \to 0-} \frac{1 - 2x^2}{-\sqrt{1 - x^2}} = \underline{\underline{-1}}$$

$\Rightarrow |x| = -x$

graf :



$H(f) = \langle 0, \frac{1}{2} \rangle$

* druhá derivace :

$$f'(x) = \frac{x - 2x^3}{\sqrt{x^2 - x^4}}$$

$$f''(x) = \frac{(1-6x^2)\cdot\sqrt{x^2-x^4} - (x-2x^3)\cdot\frac{1}{2\cdot\sqrt{x^2-x^4}}\cdot(2x-4x^3)}{\left(\sqrt{x^2-x^4}\right)^2} =$$

$$= \frac{\frac{1}{2\cdot\sqrt{x^2-x^4}}\cdot\left[2\cdot(1-6x^2)\cdot(x^2-x^4) - (x-2x^3)\cdot(2x-4x^3)\right]}{x^2-x^4} =$$

$$= \frac{\cancel{2x^2} - 12x^4 - 2x^4 + 12x^6 - \cancel{2x^2} + 4x^4 + 4x^4 - 8x^6}{2\cdot\sqrt{x^2-x^4}\cdot x^2\cdot(1-x^2)} =$$

$$= \frac{\cancel{2x^2}\cdot(-6x^2 - x^2 + 6x^4 + 2x^2 + 2x^2 - 4x^4)}{\cancel{2}\cdot\sqrt{x^2-x^4}\cdot\cancel{x^2}\cdot(1-x^2)} = \frac{-3x^2 + 2x^4}{\sqrt{x^2-x^4}\cdot(1-x^2)} =$$

$$= \frac{\cancel{x^2}\cdot(2x^2-3)\cdot\sqrt{x^2-x^4}}{(x^2-x^4)\cdot(1-x^2)} = \frac{(2x^2-3)\cdot\sqrt{x^2-x^4}}{(1-x^2)^2}$$

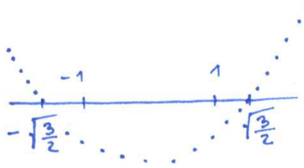
$\cancel{x^2}(1-x^2)$

$>0 \quad \forall x \in D(f'')$

(kontrola ve Wolframu - ✓)

$D(f'') = (-1, 1) \setminus \{0\}$

$2x^2 - 3$:



$\Rightarrow \quad 2x^2 - 3 < 0 \quad \forall x \in D(f'')$

$\Rightarrow \quad f''(x) < 0 \quad \forall x \in D(f'')$